INTERNATIONAL JOURNAL OF NATURAL AND APPLIED SCIENCES (IJNAS), VOL. 5, NOS.1& 2 (2010); P. 53 – 58, 3 TABLE

On determination of optimal production control using linear programming model

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ABSTRACT

The problem of minimizing cost in order to maximize profit in any establishment or business organization has been an ancient problem to many statisticians, many has proposed many solutions ranging from George Dantzig (1947) to of recent, the extension of quadratic programming by Mccarl and Spreen (2004). In this paper, linear programming model has been used to trace a long standing problem of minimizing the cost of producing flour in order to maximize daily profit in Niger Mills Company, Nigeria. From our findings, it was established that minimizing the cost of producing flour will attract a daily profit of N4,560.00 to the Company. Numerical application confirms this assertion.

INTRODUCTION

According to Garvin (1960), linear programming had its inception from within the very practical field of military logistics. It was developed by George B. Dantzig in 1947 to assist in over-all planning of the multitude of activities of the United States Air Force. The aim was to determine an optimal program of activities (a program whose "value" was an extremum), and this account for the origin of the term *Linear programming.* It soon became clear that linear programming had applications which went far beyond the military sphere. Charnes and Cooper (1953) pioneered the application of linear programming to industrial problems of planning and production; Kuhn and Tucker (1950), began to investigate important connections between linear programming and the theory of games. Hundreds of papers (both theoretical and applied) have been published in the general field of linear programming during the years, and the end of new practical applications seems not yet in sight. From a physical point of view, the field of linear programming is concerned with the optimum operation of a system of interdependent activities.

Dantzig (1947), developed a systematic procedure for solving linear programming problems. This procedure, the *Simplex Method*, came to be recognized as the most effective general method for handling linear programming problems. Subsequent research went into painstaking efforts to improve and refine the simplex method to make it a precise and reliable mathematical tool. As an iterative procedure, it was readily adopted for use on modern high-speed electronic computers. It became an important technique in theoretical investigations, and its applications expanded into industry, agriculture, transportation, economic theory, and engineering etc.

Theoretical background

Garvin (1960), opined that, soon after linear programming began to be applied to practical problems, that the computational procedure described above can be very time-consuming even when considered on the time scale of the electronic digital computers. One disadvantage of the original simplex method lies in the fact that we compute and record many numbers which either are never needed at all in the process or are needed only in an indirect way. We are forced to do this because we do not know ahead of time which numbers are needed and which numbers are superfluous.

The Revised Simplex Method explicitly uses matrix manipulations. So it is necessary to describe the problem in matrix notation. The iteration steps of the revised simplex method are exactly the same as in the tableau simplex method discussed above. The main difference is that, the computations in the revised simplex method are based on matrix algebra rather than on row operations. The use of matrix algebra reduces the adverse effect of machine round-off error by controlling the accuracy of computing the inverse (B^{-1}) of the basis matrix B.

Again, consider the general LP (maximization) problem which is written as:Maximize $f = c_1x_1 + c_2x_2 + \ldots + c_nx_n$, (1)

Subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\le b_2 \\ . \quad . \quad . \quad . & \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\le b_m \end{aligned} \tag{2}$$

and

$$x_1 \ge 0, \quad x_2 \ge 0, \ldots, x_n \ge 0 \tag{3}$$

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Using matrices, our standard form for the general LP model is given as

Maximize $f = CX$ (4)

Subject to $AX \leq b$, (5)

and $X \geq 0$. (6)

where C is the row vector: $C_{1xn} = [c_1, c_2, \ldots, c_n]$

X, and b are the column vectors such that

$$X_{nx1} = \begin{pmatrix} x_1, \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix} \quad b_{mx1} = \begin{pmatrix} b_1 \\ b_2 \\ ., \\ . \\ . \\ b_m \end{pmatrix}$$

and A is the matrix

$$A_{mxn} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ . & & & \\ . & & & \\ . & & & \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

To obtain the augmented form of the problem, i.e. by introducing the column vector of slack variables X_s

$$X_s = \begin{pmatrix} x_{n+1} \\ x_{n+2} \\ . \\ . \\ x_{n+m} \end{pmatrix}$$

So that the constraints becomes

$$[A, I] \begin{pmatrix} X \\ X_s \end{pmatrix} = b, \text{ and } \begin{pmatrix} X \\ X_s \end{pmatrix} \geq 0$$

Where I is the mxm identity matrix. (Hillier and Liebaman 1990).

Problem formulation

To formulate the mathematical model for this problem, let x_1 be the quantity to be produced of flour, x_2 be the quantity to be produced of semovita and x_3 be the quantity to be produced of wheat offal respectively, and f is the total profit that would be made from the sales of these three items. Thus, x_1, x_2 and x_3 are the decision variables for the model. The objective is to choose the values of x_1, x_2 and x_3 so as to maximize the profit function

$f = c_1x_1 + c_2x_2 + c_3x_3$ (7)

Subject to the restrictions imposed on their values by the limited daily availability.

From the accounts department, it was learnt that the cost of producing a 50kg bag of flour is N3,090.00, while that of producing a 10kg bag of semovita is N750.00 and that of producing a 25kg bag of wheat offal is put at N420.00. Also from the sales department, it was learnt that the selling price for a bag of flour is N3,280.00, while the selling price for a bag of semovita is N900.00 and that for a bag of wheat offal is N590.00. By subtracting the direct cost of each product from the selling price, we have the net revenue (profit) of ₦190, ₦150 and ₦170 made on selling a bag of Flour, Semovita and Wheat Offals. Therefore, our profit (C_1) per unit of X_1 is N190.00, while profit (C_2) per unit of X_2 is N150.00 and profit (C_3) per unit of X_3 is N170.00. Substituting these values in (7) yields:

$f = 190x_1 + 150x_2 + 170x_3$ (8)

This is the profit function that we seek to maximize.

The first row in Appendix A implies that each unit of Flour produced would use 1/20 of wheat (raw material), because one ton of wheat consumed, produces 20 (50kg) bags of flour, and a ton (= 1000kg), while each unit of Semovita produced would use 1/100 of wheat (raw material), because one ton of wheat consumed, produces 100 (10kg) bags of Semo and finally, each unit of Wheat Offals produced would also use 1/40 of the wheat (raw material), whereas only 250tons is available per day..

This restriction is expressed mathematically by the inequality:

$1/20X_1 + 1/100X_2 + 1/40\,X_3 \leq 250$ (9)

Similarly, it took 60minutes to clean and condition 250tons of wheat before milling; we note that this cleaning and conditioning process is done by machines, and these machines run for a maximum of 1440 minutes per day. This also imposes the restriction that:

$60X_1 + 60X_2 + 60\,X_3 \leq 1440$ (10)

At the milling stage, it took 15minutes to produce semovita, 20minutes to produce flour and 25minutes to produce wheat offals. This restriction is expressed mathematically by the inequality:

$20X_1 + 15X_2 + 25\,X_3 \leq 1440$ (11)

The final stage of production process is the Scaling/Packing stage. This takes 15minutes to scaled/packed Flour into 50kg bag, 10minutes to scale Wheat Offals into 25kg and 4minutes to scale and packed semovita into its 10kg bag, and leads to the restriction that:

$15X_1 + 10X_2 + 4X_3 \leq 1440$ (12)

The maximum daily demand for each product; Flour, Semovita and Wheat Offals is 3,566(50kg) bags, 1,349(10kg) bags and 1,134(25kg) bags. Therefore, the mathematical statement of these demands for Flour, Semovita and Wheat Offals is:

$X_1 \leq 3,566$ (13)

$X_2 \leq 1,349$ (14)

$X_3 \leq 1,134$ (15)

Finally, it was also observed from the account Department that the difference between the daily demand for flour(X_1) and Semo(X_2) is 2217, while that between flour(X_1) and W/Offals(X_3) is 2432 and that of Semo(X_2) and W/Offals is 215. This can be express mathematically as:

$X_1 - X_2 \leq 2217$ (16)

$X_2 - X_3 \leq 215$ (17)

$X_1 - X_3 \leq 2432$ (18)

Furthermore, in any feasible production program, the quantities produced cannot be negative; hence we have the following three inequalities **2.2**

$X_1 \geq 0, \quad X_2 \geq 0, \quad X_3 \geq 0$ (19)

The problem can now be formulated in mathematical terms as that of finding values of X_1, X_2 and X_3 which satisfy the inequalities (9), (10), (11), (12), (13), (14), (15), (16), (17), (18) and (19) and for which f as given by (8) is maximized. i.e. we seek to find values of X_1, X_2 and X_3 so as to

Maximize $f = 190X_1 + 150X_2 + 170X_3$ (20)

Subject to

$$\left.\begin{array}{ll} 1/20X_1 + 1/100X_2 + 1/40X_3 & \leq 250 \\ 60X_1 + 60X_2 + 60X_3 & \leq 1440 \\ 20X_1 + 15X_2 + 25X_3 & \leq 1440 \\ 15X_1 + 10X_2 + 4X_3 & \leq 1440 \\ X_1 - X_2 & \leq 2217 \\ X_2 - X_3 & \leq 215 \\ X_1 - X_3 & \leq 2432 \\ X_1 & \leq 3566 \\ X_2 & \leq 1349 \\ X_3 & \leq 1134 \end{array}\right\} \quad (21)$$

$X_j \geq 0, \ j = 1, 2, 3.$ (22)

This problem has all the characteristics of a linear programming problem; in a general linear programming problem, a linear function [such as (20)] is maximized subject to a number of linear inequalities and equations [such as (21 and 22)]. Therefore, it is a linear programming problem.

Table 1. Data for Flour Mills Company Problem

Item	Flour	Semovita	W/Offals	Daily Availability
Raw Material (wheat in ton)	1/20	1/100	1/40	250
Conditioning Stage	60	60	60	1440
Milling Stage	20	15	25	1440
Scaling/Packing Stage	15	10	4	1440
Difference in Demand b/w Flour & Semo	1	-1	-	2217
Difference in Demand b/w Semo & W/Offals	-	1	-1	215
Difference in Demand b/w Flour & W/Offals	1	-	-1	2432
Demand for Flour	1	-	-	3566
Demand for Semovita	-	1	-	1349
Demand for W/Offals	-	-	1	1134
Net Revenue (₦/Unit)	190	150	170	

Linear programming solution by excel solver

Table 2. Flour Mill Model

The problem data are inputted into the system as shown in table 2

FLOUR MILL MODEL						
Input Data						
	X1	X2	X3			
	Flour	**Semovita**	**W/Offals**	**TOTALS**		**LIMITS**
Objective	190	150	170	4560		
Raw Material	1/20	1/100	1/40	1.2	<=	250
Time 1	60	60	60	1440	<=	1440
Time 2	20	15	25	480	<=	1440
Time 3	15	10	4	360	<=	1440
Demand 1	1	-1	0	24	<=	2217
Demand 2	0	1	-1	0	<=	215
Demand 3	1	0	-1	24	<=	2432
Demand 4	1	0	0	24	<=	3566
Demand 5	0	1	0	0	<=	1349
Demand 6	0	0	1	0	<=	1134
	>=0	>=0	>=0			
Output Results:						
	X1	X2	X3	*F*		
Solution	**24**	**0**	**0**	**4560**		

Table 3. Solver's output summary

Microsoft Excel 11.0 Answer Report

Worksheet: [Flour Mill Model.xls]Sheet1

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Target Cell (Max)

Cell	Name	Original Value	Final Value
$E$5	Objective TOTALS	0	4560

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$19	Solution X1	0	24
$C$19	Solution X2	0	0
$D$19	Solution X3	0	0

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$6	Raw Material TOTALS	1.2	$E$6<=$G$6	Not Binding	248.8
$E$7	Time 1 TOTALS	1440	$E$7<=$G$7	Binding	0
$E$8	Time 2 TOTALS	480	$E$8<=$G$8	Not Binding	960
$E$9	Time 3 TOTALS	360	$E$9<=$G$9	Not Binding	1080
$E$10	Demand 1 TOTALS	24	$E$10<=$G$10	Not Binding	2193
$E$11	Demand 2 TOTALS	0	$E$11<=$G$11	Not Binding	215
$E$12	Demand 3 TOTALS	24	$E$12<=$G$12	Not Binding	2408
$E$13	Demand 4 TOTALS	24	$E$13<=$G$13	Not Binding	3542
$E$14	Demand 5 TOTALS	0	$E$14<=$G$14	Not Binding	1349
$E$15	Demand 6 TOTALS	0	$E$15<=$G$15	Not Binding	1134
$B$19	Solution X1	24	$B$19>=0	Not Binding	24
$C$19	Solution X2	0	$C$19>=0	Binding	0
$D$19	Solution X3	0	$D$19>=0	Binding	0

RESULT

The Excel solver output summary is in three sections, namely the target cell section showing the optimum value of the objective function (*f*), the adjustable cells section which reveals the values of the decision variables (X_1, X_2, and X_3) and the constraints section which shows the values of the slack variables (X_4, X_5, X_6, X_7, X_8, X_9, X_{10} X_{11}, X_{12}, and X_{13}).

From the target cell section of the Excel Solver's output above, it was observed that the maximum daily profit (*f*) that would be made by the company is N4,560.00. This value is achieved when Niger Mills Company produces 24(50kg) bags of Flour (X_1) per day.

Except for the first time constraint, all the values of slack variables at the optimum are positive, and this means that, the constraints are all satisfied and the resources (raw material, production time and demand) are in abundance.

CONCLUSION

In this paper, we were able to identify the problem of Flour Mills Company as a linear programming problem, formulate a mathematical model that represents the essence of the problem, identify the functional constraints of the problem as well as solved the problem using the available computer software (Excel Solver) for solving linear programming problems. From the solution obtained, we were also able to determine an optimal production control for the Company.

ACKNOWLEDGEMENT

We thank the Editor and the reviewers for their insightful comments that enhanced the paper.

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